Mathematics for Economics and Finance

Preparation

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Modern economics and finance rely profoundly on mathematics. It is therefore essential for incoming M.Sc. students to be familiar with some basic concepts of mathematics used in economics and finance applications. When starting their graduate studies, students should have a working knowledge of the material covered in the following basic textbook:

- Simon, C. P. and L. Blume, 1994, Mathematics for Economists, Norton, New York.

There are numerous advanced textbooks on the mathematics for economics and finance. No single book is comprehensive. The advanced textbooks recommended for the course are as follows:

- Sydsaeter, K. , P. Hammond, A. Seierstad, and A. Strom, 2005, Further Mathematics for Economic Analysis, Prentice Hall.

- Casella, G. and R. L. Berger, 2001, Statistical Inference, 2nd edition, Brooks Cole.

In order to refresh and broaden your knowledge, working through some exercises before the semester is a useful preparation. I strongly encourage all incoming M.Sc. students to prepare written solutions to the following set of exercises (at varying levels of difficulty).

Exercise 1 (Logic)

Look at the following pairs of statements and determine if they are logically equivalent. p*,* q*, and* r *are mathematical statements.*

(a) $\neg(p \wedge q)$ *and* $(\neg p) \vee (\neg q)$.

(b) $(p \Rightarrow q) \wedge (q \Rightarrow r)$ *and* $p \Rightarrow r$.

Exercise 2 (Proofs)

Show by Mathematical Induction that for all n *and for all* $x \neq 1$, $1 + x + ... + x^n = \frac{1-x^{n+1}}{1-x}$.

Exercise 3 (Topology)

The consumption set of a consumer is

$$C = \left\{(x, y) \in R_+^2 : x \geqslant x' > 0,\ y \geqslant y' > 0\right\}.$$

Illustrate this set. Is it closed? Is it bounded? What is the interior of C*? How would you interpret* x' *and* y'*?*

Exercise 4 (Matrix Algebra)

You have just finished your master's studies at HEC with success. Not surprisingly, you have been hired as an investment advisor for OBS (Ordinary Bank of Switzerland). You first client is a wealthy Saudi-Arabian who wants you to do some asset allocation. The asset classes he is considering are bonds, stocks, and real estate. You have done some preliminary analysis and determined that the payoffs of the three types of securities in three possible scenarios are

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -3 & 2 \\ 2 & -2 & 3 \end{bmatrix}$$

where each column is a different asset, and each row is a different scenario. Your goal is *to determine the portfolio allocation in each asset class, denoted $\theta \in \mathbb{R}^3$, given the client's target payoff $b \in \mathbb{R}^3$ in each scenario. In other words, you want to determine θ such that $A\theta = b$. To make sure that this is feasible, you determine the following:*

(a) *Calculate the determinant $|A|$ of A.*

(b) *Is A invertible? Justify briefly.*

(c) *Calculate the inverse matrix A^{-1}.*

(d) *What is the optimal portfolio θ given $b = (b_1; b_2; b_3)'$?*

Exercise 5 (Limits)

Compute the limit of the following functions:

(a) $\lim_{x\to 0} \left[x^2 \sin\left(\frac{1}{x}\right)\right]$

(b) $\lim_{x\to 0} \frac{\sin^2 x}{x}$

Exercise 6 (Derivatives)

Compute the derivatives of the following functions:

(a) $y = \ln(x + \sqrt{1 + x^2})$

(b) $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$.

Exercise 7 (Concavity)

Let $f(x) = x^3$ and $g(y) = -y$. Check whether f, g are strictly concave, strictly convex, concave, convex, pseudo-concave, pseudo-convex, quasi-concave, quasi-convex.

Exercise 8 (Taylor Approximation)

Consider the function $f(x) = \ln(x^2 + x - 1)$. Compute the exact value of f at $x = 1.1$ and compare it to the third-order Taylor approximation in the neighborhood of $x = 1$. What is the order of the approximation error?

Exercise 9 (Integrals)

Evaluate the following integrals:

(a) $\int \frac{dx}{1+3\sqrt{x-1}}$

(b) $\int_0^1 e^{\sqrt{x}} dx$

Exercise 10 (Probability)

Two students John and Mary are taking a mathematics course. The course has only three grades: A, B, and C. The probability that John gets a B is 30%. The probability that Mary gets a B is 40%. The probability that neither gets an A but at least one gets a B is 10%. What is the probability that at least one gets a B but neither gets a C?

Exercise 11 (Statistics)

Many quantitative models of the stock market assume that stock returns follow a log-normal distribution. That is, $\ln(X) \sim N(\mu, \sigma^2)$, $0 < X < \infty$, $-\infty < \mu < \infty$, $\sigma^2 > 0$.

(a) *Find the probability density function for* X.

(b) *Compute the expectation and variance of* X, $E(X)$ *and* $Var(X)$.

Exercise 12 (Static Optimization)

An agent faces the following optimization problem:

$$\begin{aligned} \max_{(x,y,z)\in \boldsymbol{D}} f(x,y,z) &= xy + 2xz + 2yz, \\ s.t.\ \boldsymbol{D} &= \{(x,y,z) \in R^3 \mid 2z^2 = x + y,\ x^2 + y^2 \leq 2\}. \end{aligned}$$

(a) *Explain why* f *must have a maximum on* $\boldsymbol{D}$.

(b) *Check for which points the constraint qualifications are satisfied.*

(c) *Formulate the Kuhn-Tucker equations that provide information about the maxima of* f *on* $\boldsymbol{D}$.

(d) *Solve the Kuhn-Tucker equations to find the solution to the optimization problem.*

Exercise 13 (Dynamic Optimization)

Suppose you have a cake of size W_1, *and the cake doesn't depreciate (melt) or grow. At each point in time,* $t = 1, 2, 3, ...T$, *you can consume some of the cake. Let* c_t *be your consumption in period* t *and* $u(c_t)$ *represent the flow of utility from this consumption. The lifetime utility is defined by* $\sum_{t=1}^{T} \beta^{t-1} u(c_t)$, *where* $0 \leq \beta \leq 1$. *The transition equation for the cake is given by* $W_{t+1} = W_t - c_t$. *Assume* $u(c) = \ln c$. *How would you find the optimal path of consumption* $\{c_t\}_{t=1}^{T}$? *What if you let* $T \to \infty$?